# QCD SUM RULES CALCULATION OF THE SINGLET AXIAL CONSTANT

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#### **Abstract**

We analyze the singlet axial form factor of the proton for small momentum transferred in the framework of QCD sum rules using the interpolating nucleon current which explicitly accounts for the gluonic degrees of freedom. As the result we come to the quantitative description of the singlet axial constant.

#### 1. Introduction.

The investigation of polarized deep inelastic scattering is one of the most attractive field for theoretical consideration since it provides an important insight into the structure of hadrons and opens a large area of subtle dynamical phenomena associated with the spin dependent case. In the last several years there has been an increasing interest in the deep inelastic structure function  $g_1^p(x)$ . It was provoked by the EMC result on the scattering of the longitudinally polarized muon beam on a longitudinally polarized hadron target. The unexpectedly small asymmetry found by EMC has led to the so called "spin crisis in the parton model" and has raised a number of questions of understanding the dynamics of the proton spin on the parton level, namely, how the nucleon spin is build up from the spins of its constituents. An enormous flood of theoretical investigations was generated in order to resolve the current "spin problem" [1].

The EMC measurement of the first moment of the polarized structure function  $\Gamma_1^p$  can be interpreted, via the Ellis-Jaffe sum rule [2]

$$\Gamma_1^p(\mathcal{Q}^2) = \int_0^1 dx g_1^p(x, \mathcal{Q}^2) 
= \frac{1}{12} \left\{ \left( G_A^{(3)}(0) + \frac{1}{\sqrt{3}} G_A^{(8)}(0) \right) \left( 1 - \left( \frac{\alpha_s}{\pi} \right) - 3.5833 \left( \frac{\alpha_s}{\pi} \right)^2 - 20.2153 \left( \frac{\alpha_s}{\pi} \right)^3 \right) 
+ \frac{4}{3} G_A^{(0)}(0, \mathcal{Q}^2) \left( 1 - \frac{1}{3} \left( \frac{\alpha_s}{\pi} \right) - 1.0959 \left( \frac{\alpha_s}{\pi} \right)^2 \right) \right\},$$
(1)

as a first ever measurement of the singlet axial constant  $G_A^{(0)}(0)$  of the proton. The last one turns out to be unexpectedly small in contradiction with the naïve parton model where it is fairly close to unity. The EMC reported the result for  $G_A^{(0)}(0)$  which is compatible with zero. The new experiments are performed to check their measurement of  $g_1^p(x)$  and to measure an analogous neutron function  $g_1^n(x)$ . The recent analysis [3] of proton and deuteron data gives  $G_A^{(0)}(0)$  varying from  $0.20 \pm 0.11$  to  $0.36 \pm 0.05$ , that is still far from unity. So the problem reduces to the evaluation of  $G_A^{(0)}(0)$  because the other two axial constants can be extracted reliably from the data on deuteron (or  $He^3$ ) target and on neutron and hyperon  $\beta$ -decays. In this paper we calculate it in the framework of QCD sum rule approach which till now seems to be the most powerful method for extraction of information about the low energy properties of hadrons and the closest one to the first principles of the theory.

In eq.(1) the functions  $G_A^{(i)}(Q^2)$  are form factors at zero momentum transferred in the proton matrix elements of axial currents

$$\langle N(p_2, \lambda_2) | j_{5\mu}^{(i)}(0) | N(p_1, \lambda_1) \rangle$$

$$= \bar{u}_N^{(\lambda_2)}(p_2) \left( G_A^{(i)}(Q^2) \gamma_\mu \gamma_5 - G_P^{(i)}(Q^2) q_\mu \gamma_5 \right) u_N^{(\lambda_1)}(p_1), \quad (2)$$

where i is a  $SU(3)_f$  index,  $q = p_2 - p_1$  and  $Q^2 = -q^2$ . There is an important difference in the behaviour of induced pseudoscalar form factors at small momentum q. Here, the singlet pseudoscalar form factor does not acquire a Goldstone pole at  $Q^2 = 0$ , even in the chiral limit, contrary to the matrix elements of the octet currents. It is known that this limit, in which the masses of the three light quark flavours are neglected, is not far away from the real world of hadrons. In this limit, there exist eight massless pseudoscalar mesons serving as Goldstone bosons. However, the ninth pseudoscalar, the  $\eta'$ -meson, remains massive. In the following this property will be used to extract a value of  $G_A^{(0)}(0)$  from the sum rules.

It has been established [1] that the first moment  $\Gamma_1^p$  does not measure the contribution of the quark spins to the proton one. This happens due to the anomalous nonconservation of the singlet axial current. For this reason, we display the rôle of this profound feature of the theory from the very beginning exploiting the equation for the anomalous divergence<sup>1</sup>:

$$\partial_{\mu} j_{5\mu}^{(0)} = 2i \sum_{q} m_{q} \bar{q} \gamma_{5} q - \left(\frac{N_{f} \alpha_{s}}{4\pi}\right) G_{\mu\nu}^{a} \widetilde{G}_{\mu\nu}^{a}, \tag{3}$$

where  $N_f$  is a number of flavours (later  $N_f = 3$ ). Taking the divergence of eq.(2) for the singlet axial current and making use of the last expression we come to the relation which directly connects, in the chiral, limit the nonforward matrix elements of the gluon operator to the effective form factor  $2m_N G_{eff}^{(0)}(Q^2) = 2m_N G_A^{(0)}(Q^2) + Q^2 G_P^{(0)}(Q^2)$  that is equal to the  $2m_N G_A^{(0)}(0)$  at  $Q^2 = 0$ .

<sup>&</sup>lt;sup>1</sup>Throughout the paper, we adopt the conventions in Itzykson and Zuber [4].

#### 2. Effective axial form factor in QCD sum rules.

For a long time all calculations of the nucleon characteristics use a particular three-quark current introduced by Ioffe [5]. When one makes an attempt to evaluate the matrix elements of quark-gluon or gluon operators, one faces evident calculational difficulties, moreover, the final sum rules are aggravated by extra UV logarithms due to mixing of operators and, therefore, the calculations are affected by noncontrollable uncertainties [6].

In field theory, the usual statement that the nucleon mainly consists of three quarks means that the  $\beta$  quarks  $\rightarrow$   $\beta$  quarks Green function (three quarks are in a state with nucleon quantum numbers) has a pole at the mass of the nucleon, with the total angular momentum  $J=\frac{1}{2}$ , with a significant residue. The fact that the nucleon is not just a three quark state means that the nucleon pole also occurs, albeit with smaller residue, in Green functions such as  $\beta$  quarks+ $g \rightarrow \beta$  quarks+g. For this reason, one is forced to introduce a more sophisticated interpolating proton field which explicitly contains the gluonic degrees of freedom:

$$\eta_G(x) = \epsilon^{ijk} \left( u^i(x) C \gamma_\mu u^j(x) \right) \gamma_5 \gamma_\mu \sigma_{\alpha\beta} \left( g G^a_{\alpha\beta}(x) t^a d(x) \right)^k. \tag{4}$$

The latter was investigated in ref. [7] and checked in the calculation of proton gluonic form factor normalized to the fraction of nucleon momentum carried by gluons. Recently, making use of this current the twist-3 and twist-4 corrections to Bjorken and Ellis-Jaffe sum rule have been found [8]. The advantages of this current are straightforward: the calculations are drastically simplified, sum rules are free from additional divergences that are not removed by the single Borel transformation. At the same time, the applicability of non-dimensional regularization may be spoiled by the power UV divergencies, appearing due to the high mass dimnension of this current. For the same reason, sum rules may be affected by the vacuum condensates of higher dimensions, reducing their outcome to semiquantitative estimates.

The usual technique of QCD sum rules is to extract the nucleon matrix element of local operator from the appropriate three-point correlation function. This correlator is the sum of different tensor structures each characterized by the relevant invariant amplitude  $W^{(i)}(p_1^2, p_2^2, q^2)$ .

$$W(p_{1}, p_{2}, q) = i^{2} \int d^{4}x d^{4}y e^{ip_{1}x - ip_{2}y} \langle 0|T \left\{ \eta_{G}(x) \left( \frac{N_{f}\alpha_{s}}{4\pi} \right) G_{\mu\nu}^{a}(0) \widetilde{G}_{\mu\nu}^{a}(0) \overline{\eta}_{G}(y) \right\} |0\rangle$$

$$= \sigma_{\mu\nu} \gamma_{5} p_{1\mu} p_{2\nu} W^{(1)}(p_{1}^{2}, p_{2}^{2}, q^{2}) + \mathcal{Q}\gamma_{5} W^{(2)}(p_{1}^{2}, p_{2}^{2}, q^{2}) + q^{2} \gamma_{5} W^{(3)}(p_{1}^{2}, p_{2}^{2}, q^{2}).$$
(5)

In practical calculation it is advantageous to consider  $W^{(1)}(p_1^2, p_2^2, q^2)$  (hereafter referred to as W) because of its lower dimensionality. Another reason in favour of this choice is that it does not lead to the fictitious kinematical singularities in  $q^2$  as the last term in eq.(5) does. For this invariant amplitude we can write the double dispersion representation

$$W(p_1^2, p_2^2, q^2) = \frac{1}{\pi^2} \int_0^\infty \int_0^\infty ds_1 ds_2 \frac{\rho(s_1, s_2, Q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)} + \cdots,$$
 (6)

where the ellipses stand for the polynomials in  $p_1^2$  and  $p_2^2$  which die out after the double Borel transformation has been applied. For the physical spectral density we have accepted the conventional "resonance plus continuum" model:

$$\rho(s_1, s_2, Q^2) = \pi^2 m_N^4 \lambda_G^2 2m_N G_{eff}^{(0)}(Q^2) \delta(s_1 - m_N^2) \delta(s_2 - m_N^2) + \rho_{cont}(s_1, s_2, Q^2) (1 - \theta(\sigma_0 - s_1)\theta(\sigma_0 - s_2)).$$
 (7)

The function in front of the double-pole term is a combination of form factors we are interested in up to certain overlap  $\lambda_G$  between the state created from the vacuum by  $\eta_G$  and the nucleon state

$$\langle 0|\eta_G(0)|N(p,\lambda)\rangle = m_N^2 \lambda_G u_N^{(\lambda)}(p). \tag{8}$$

So, our aim is the evaluation of the correlation function (5) in QCD. In the case when all the momenta  $(-p_1^2) \sim (-p_2^2) \sim Q^2$  are sufficiently large (of an order of  $1 GeV^2$ ), the leading contribution comes from the domain where all distances are small. Thus, the standard machinery of short distance expansion are applicable, allowing one to express the final result in terms of quark and gluon condensates. The problem is modified drastically if the squared momentum transferred becomes small  $(Q^2 \ll (-p_i^2))$  because the relevant t-channel distances can be large. In this case the OPE has a twofold structure [9]. Terms of the first type arise from the SD(I)-region when all intervals  $x^2 \sim y^2 \sim (x-y)^2$  are small. Another contribution comes from SD(II)-region (bilocal power correction) which originates from

the distances  $x^2 \sim y^2 \gg (x-y)^2$ . The necessity for the bilocal power corrections can be traced from the fact that the ordinary QCD Feynman diagrams contributing to the form factor at moderately large  $Q^2$  in the limit of small  $Q^2$  possess logarithmic non-analyticities  $(Q^2)^n ln Q^2$ , which signals that large distances come into play [10]. Therefore, we have to subtract such a perturbative behaviour from the corresponding graphs and add the "exact" correlators which account for the nonperturbative effects and thus possess the correct analytical properties as  $Q^2$  goes to zero. So the OPE in the case when the momentum transferred can be arbitrary small has a modified form [9, 11]

$$W(p_1^2, p_2^2, q^2) = \sum_{d} C_{SD(I)}^{(d)}(p_1^2, p_2^2, q^2) \langle \mathcal{O}_d \rangle + \sum_{i} \int d^4 x e^{ipx} C_{SD(II)}^{(i)}(x^2) W_i^{BL}(x, q), \quad (9)$$

where, as was mentioned above, the coefficients  $C_{SD(I)}^{(d)}(p_1^2, p_2^2, q^2)$  are regular in the limit  $Q^2 \to 0$ . The second term determines the large t-distance contribution. Here  $W_i^{BL}$  are the two-point correlators

$$W_i^{BL}(x,q) = \int d^4 y e^{iqx} \langle 0|T \left\{ \mathcal{G}(y)\mathcal{O}_i(x,0) \right\} |0\rangle$$
 (10)

of operator in question  $\mathcal{G}(y) = \left(\frac{N_f \alpha_s}{4\pi}\right) G_{\mu\nu}^a(y) \widetilde{G}_{\mu\nu}^a(y)$  and some nonlocal string operator with definite twist (not dimension) [12, 13] that arises from the OPE of the T-product of nucleon currents:

$$T\{\eta_G(x)\bar{\eta}_G(0)\} = \sum_i C_{SD(II)}^{(i)}(x^2)\mathcal{O}_i(x,0)$$
 (11)

The bilocal power corrections cannot be directly calculated in perturbation theory but we can write down the dispersion relation for them

$$W_i^{BL}(x,q) = \frac{1}{\pi} \int_0^\infty ds \frac{\rho_i(s,(xq),x^2)}{s-q^2},$$
 (12)

assuming the standard spectral density model with continuum to start at some threshold  $s_0$  and finding in some way its parameters. We always do this constructing auxiliary sum rules. There is no need in additional subtractions in eq.(12) because one always deals with the difference between the "exact" bilocal and its perturbative part; so due to the coincidence of their UV behaviours the subtraction terms cancel in this difference.

To simplify the calculation of the local power corrections, it is convenient to use fixed-point gauge for the background gluon field  $(x-x_0)_{\mu}B^a_{\mu}(x)=0$ .

We chose the fixed point in the vertex of the gluon operator  $x_0 = 0$ . The quark and gluon propagators in this gauge up to the order O(G) looks like [14]

$$D_{\mu\nu}^{ab}(x,y) = -i\frac{\Gamma(\frac{d}{2}-1)}{4\pi^{\frac{d}{2}}} \frac{g_{\mu\nu}\delta^{ab}}{[-\Delta^{2}]^{\frac{d}{2}-1}} + 2iG_{\mu\nu}^{ab} \frac{\Gamma(\frac{d}{2}-2)}{16\pi^{\frac{d}{2}}} \frac{1}{[-\Delta^{2}]^{\frac{d}{2}-2}} + iG_{\rho\sigma}^{ab} \frac{\Gamma(\frac{d}{2}-1)}{8\pi^{\frac{d}{2}}} \frac{g_{\mu\nu}y_{\rho}x_{\sigma}}{[-\Delta^{2}]^{\frac{d}{2}-1}},$$

$$S(x,y) = \frac{\Gamma(\frac{d}{2})}{2\pi^{\frac{d}{2}}} \frac{\cancel{\Delta}}{[-\Delta^{2}]^{\frac{d}{2}}} - \widetilde{G}_{\mu\nu}\gamma_{\nu}\gamma_{5} \frac{\Gamma(\frac{d}{2}-1)}{8\pi^{\frac{d}{2}}} \frac{\Delta_{\mu}}{[-\Delta^{2}]^{\frac{d}{2}-1}} + iG_{\mu\nu}y_{\mu}x_{\nu} \frac{\Gamma(\frac{d}{2})}{4\pi^{\frac{d}{2}}} \frac{\cancel{\Delta}}{[-\Delta^{2}]^{\frac{d}{2}}}, \quad (13)$$

where  $\Delta = x - y$ ,  $G^{ab}_{\mu\nu} = gf^{acb}G^c_{\mu\nu}$  for the gluon propagator and  $G_{\mu\nu} = gt^aG^a_{\mu\nu}$  for the quark one, the generators are normalized by  $Sp(t^at^b) = \frac{1}{2}\delta^{ab}$ . For the noncollinear quark condensate we use the following expansion in terms of local vacuum expectation values [15]:

$$\langle \bar{\psi}_{\alpha}^{i}(y)\psi_{\beta}^{i}(x)\rangle = \frac{1}{4}\langle \bar{\psi}\psi\rangle I_{\beta\alpha} + \frac{1}{4^{3}}m_{0}^{2}\langle \bar{\psi}\psi\rangle \left[ (x-y)^{2} - i\frac{2}{3}\sigma_{\mu\nu}x_{\mu}y_{\nu} \right]_{\beta\alpha} + \dots$$
(14)

The ellipses stand for the higher dimension vacuum condensates.

Calculating the diagrams depicted in the first row of fig. 1 we come to the Borel sum rule for effective axial form factor at moderate values of the momentum transferred with the Borel parameters  $\tau_1$  and  $\tau_2$ :

$$\frac{m_N^4 \lambda_G^2}{\tau_1 \tau_2} 2 m_N G_{eff}^{(0)}(Q^2) e^{-\frac{m_N^2}{\tau_1} - \frac{m_N^2}{\tau_2}} 
= -\frac{1}{\pi^2} \int_0^\infty \int_0^\infty \frac{ds_1 ds_2}{\tau_1 \tau_2} (1 - \theta(\sigma_0 - s_1)\theta(\sigma_0 - s_2)) \rho_{cont}(s_1, s_2, Q^2) e^{-\frac{s_1}{\tau_1} - \frac{s_2}{\tau_2}} 
+ \frac{N_f}{\pi^2} \left(\frac{\alpha_s}{\pi}\right)^2 \langle \bar{u}u \rangle \left\{ \frac{1}{3} Q^2 \frac{(\tau_1 \tau_2)^2}{(\tau_1 + \tau_2)^3} J_{12} \right. 
+ \frac{1}{18} m_0^2 \left[ 4 \frac{(\tau_1 \tau_2)^2}{(\tau_1 + \tau_2)^3} J_{12} - Q^2 \frac{(\tau_1 \tau_2)}{(\tau_1 + \tau_2)^2} (4J_{11} - J_{02}) \right] 
+ \frac{1}{144} m_0^2 \left[ 4 \frac{(\tau_1 \tau_2)^2}{(\tau_1 + \tau_2)^3} J_{12} + Q^2 \frac{(\tau_1 \tau_2)}{(\tau_1 + \tau_2)^2} J_{02} \right] 
- \frac{7}{32} m_0^2 \frac{(\tau_1 \tau_2)^2}{(\tau_1 + \tau_2)^3} J_{12} - \frac{1}{8} m_0^2 \left[ 2 \frac{(\tau_1 \tau_2)^2}{(\tau_1 + \tau_2)^3} J_{12} - Q^2 \frac{(\tau_1 \tau_2)}{(\tau_1 + \tau_2)^2} J_{02} \right] \right\}$$
(15)

where the continuum double spectral density is

$$\rho_{cont}(s_1, s_2, Q^2) = \sum_{i=1}^{5} \rho_{(i)}(s_1, s_2, Q^2), \tag{16}$$

and each term in a sum is found from the corresponding diagram in fig.1 (appropriate formulae are given in appendix B):

$$\rho_{(1)}(s_1, s_2, Q^2) = \frac{N_f}{72} \left(\frac{\alpha_s}{\pi}\right)^2 \langle \bar{u}u \rangle Q^4 \left(1 - \frac{\sigma}{R^{\frac{1}{2}}}\right)^2 \left(2 + \frac{\sigma}{R^{\frac{1}{2}}}\right), \tag{17}$$

$$\rho_{(2)}(s_1, s_2, Q^2) = \frac{N_f}{18} \left(\frac{\alpha_s}{\pi}\right)^2 m_0^2 \langle \bar{u}u \rangle Q^2 \left\{ \frac{1}{6} \left(1 - \frac{\sigma}{R^{\frac{1}{2}}}\right)^2 \left(2 + \frac{\sigma}{R^{\frac{1}{2}}}\right) - \frac{1}{2} \left(1 - \frac{\sigma}{R^{\frac{1}{2}}}\right) - 5Q^2 \frac{s_1 s_2}{R^{\frac{3}{2}}} \right\},$$

$$(18)$$

$$\rho_{(3)}(s_1, s_2, Q^2) = \frac{N_f}{144} \left(\frac{\alpha_s}{\pi}\right)^2 m_0^2 \langle \bar{u}u \rangle Q^2 \left\{ \frac{1}{6} \left(1 - \frac{\sigma}{R^{\frac{1}{2}}}\right)^2 \left(2 + \frac{\sigma}{R^{\frac{1}{2}}}\right) - \frac{1}{2} \left(1 - \frac{\sigma}{R^{\frac{1}{2}}}\right) - Q^2 \frac{s_1 s_2}{R^{\frac{3}{2}}} \right\},$$
(19)

$$\rho_{(4)}(s_1, s_2, Q^2) = -\frac{7N_f}{768} \left(\frac{\alpha_s}{\pi}\right)^2 m_0^2 \langle \bar{u}u \rangle Q^2 \left(1 - \frac{\sigma}{R^{\frac{1}{2}}}\right)^2 \left(2 + \frac{\sigma}{R^{\frac{1}{2}}}\right), \tag{20}$$

$$\rho_{(5)}(s_1, s_2, Q^2) = -\frac{N_f}{8} \left(\frac{\alpha_s}{\pi}\right)^2 m_0^2 \langle \bar{u}u \rangle Q^2 \left\{ \frac{1}{12} \left(1 - \frac{\sigma}{R^{\frac{1}{2}}}\right)^2 \left(2 + \frac{\sigma}{R^{\frac{1}{2}}}\right) + \frac{1}{2} \left(1 - \frac{\sigma}{R^{\frac{1}{2}}}\right) + Q^2 \frac{s_1 s_2}{R^{\frac{3}{2}}} \right\},$$
(21)

and

$$\sigma = s_1 + s_2 + Q^2, \qquad R(s_1, s_2, Q^2) = \sigma^2 - 4s_1 s_2.$$
 (22)

The functions  $J_{nm}$  are originated from the diagrams in the first row in fig.1 and are given by the following expression:

$$J_{nm}(\tau_i, Q^2) = \int_0^1 dx \bar{x}^{n-1} x^m exp \left\{ -\frac{x}{\bar{x}} \frac{Q^2}{(\tau_1 + \tau_2)} \right\}$$
 (23)

We state that contrary to the refs.[8] where the sum rules with the same interpolating nucleon field were dominated by the contribution from

the highest dimension operators, our sum rule is not affected by them: the coefficient functions that are determined to the leading accuracy by tree and one-loop diagrams vanish identically. Therefore, we do not meet the problem of breakdown of OPE for the correlator in question. The absence of higher condensates contribution unsuppressed by a number of loops is directly connected with chiral structure of the interpolating nucleon field and the tensor structure chosen for investigation in the three-point correlation function.

As was mentioned previously one could not put  $Q^2 = 0$  in the eq. (15) because though finite it is including contributions non-analytic in this point. This is typical example of the mass singularities. Therefore, following the method outlined in the middle of this section one should subtract the perturbative behaviour from corresponding graphs (the diagrammatic representation for them is shown in the second row of fig. 1, while the explicit expressions are written in the appendix A) and add the terms with correct singular structure in  $Q^2$ . It is clear that the singularity should be located at the threshold of the first prominent resonance in the corresponding channel.

### 3. Bilocal corrections.

The simplest bilocal correction (first picture on fig. 2) is given by the convolution of the coefficient function involving the quark condensate with the two-point correlation function of operator in question and some point-splitted gluon operator coming from the OPE of nucleon fields

$$W^{BL}_{\lambda\kappa}(x,q) = i \left(\frac{N_f \alpha_s}{4\pi}\right)^2 \int d^4y e^{iqy} \langle 0|T \left\{G^a_{\mu\nu}(y)\widetilde{G}^a_{\mu\nu}(y) \left(G^a_{\mu\lambda}(x)\widetilde{G}^a_{\mu\kappa}(0) - G^a_{\mu\kappa}(x)\widetilde{G}^a_{\mu\lambda}(0)\right)\right\} |0\rangle$$

$$(24)$$

As was mentioned earlier we cannot calculate it in perturbation theory but we can reconstruct it from the information about its large- $Q^2$  behaviour. To this end we write down the dispersion relation for it of the type represented by eq. (12) and use the standard "resonance plus continuum" spectral density model, with  $\eta'$ -meson. It is likely to be the only prominent *singlet* pseudoscalar both in quark and gluon channels.

$$\rho(s) = \pi m_{\eta'}^2 f_{\eta'} [i f_{\eta'}^{(1)} \phi_{\eta'}^{(1)}(xq) + f_{\eta'}^{(2)}(xq) \phi_{\eta'}^{(2)}(xq)] \delta(s - m_{\eta'}^2) + \theta(s - s_0) \rho^{PT}(s).$$
(25)

Here we have parametrized the matrix elements of some gluon operators between the vacuum and  $\eta'$ -meson state as follows:

$$\langle 0 | \frac{N_f \alpha_s}{4\pi} G_{\mu\nu}^a(0) \widetilde{G}_{\mu\nu}^a(0) | \eta'(q) \rangle = m_{\eta'}^2 f_{\eta'}$$

$$\langle \eta'(q) | \frac{N_f \alpha_s}{4\pi} (G_{\mu\lambda}^a(x) \widetilde{G}_{\mu\kappa}^a(0) - G_{\mu\kappa}^a(x) \widetilde{G}_{\mu\lambda}^a(0)) | 0 \rangle$$

$$= (x_{\kappa} q_{\lambda} - x_{\lambda} q_{\kappa}) [i f_{\eta'}^{(1)} \phi_{\eta'}^{(1)}(xq) + f_{\eta'}^{(2)}(xq) \phi_{\eta'}^{(2)}(xq)]. \tag{26}$$

In the last line the wave functions  $\phi_{\eta'}^{(i)}$  can be related in the standard way to the usual ones  $(\varphi^{(i)})$ , describing the light-cone momentum fraction distribution of gluon inside meson.

$$\phi^{(i)}(xq) = \int_0^1 d\alpha e^{i\alpha(xq)} \varphi^{(i)}(\alpha). \tag{27}$$

In eq. (26) we have kept only the leading twist wave functions which reproduce the leading nonanalyticity in the corresponding contribution of the local  $\langle \bar{u}u \rangle$  power correction. As will be shown below, we account for other contribution using simple recipe which results from our consideration (originally it was proposed in Ref. [16]). We can find the unknown overlaps  $f_{\eta'}^{(i)}$  constructing the auxiliary sum rules. It turns out, that due to the antisymmetrical tensor structure involved the contribution of ordinary local power corrections with gluon condensates, unsuppressed by a loop factor, vanish identically in the theoretical part of the sum rules. For this reason we account for nonperturbative effects introducing the concept of nonlocal gluon condensate [17] which corresponds to infinite series of local ones. It can be appropriately decomposed into two tensor structures multiplied by corresponding form factors [15, 18]:

$$\langle 0|G_{\mu\rho}^{a}(x)\widetilde{E}^{ab}(x,0)G_{\nu\sigma}^{b}(0)|0\rangle = \frac{\langle G^{2}\rangle}{12} \Big\{ (g_{\mu\nu}g_{\rho\sigma} - g_{\mu\sigma}g_{\nu\rho}) \left( D_{NA}(x^{2}) + D_{A}(x^{2}) \right) + (g_{\mu\nu}x_{\rho}x_{\sigma} + g_{\rho\sigma}x_{\mu}x_{\nu} - g_{\mu\sigma}x_{\nu}x_{\rho} - g_{\nu\rho}x_{\mu}x_{\sigma}) \frac{dD_{A}(x^{2})}{dx^{2}} \Big\}.$$
 (28)

This form explicitly separates out the term proportional to  $D_{NA}(x^2)$  which violate the abelian Bianchi identity, while the second term satisfies it. It

was shown that linear confinement occurs when  $D_{NA}(x^2)$  is present in (28) while the second term does not contribute to the string tension [18].

In the calculation of  $f_{\eta'}^{(1)}$  and  $f_{\eta'}^{(2)}$  constants only abelian form factor contribute. We present it in the form of  $\alpha$ -representation [19]:

$$D_A(x^2) = \int_0^\infty d\alpha f_G^A(\alpha, \lambda_A^2) e^{\alpha \frac{x^2}{4}}.$$
 (29)

and use a  $\delta$ -shaped ansatz for the distribution function  $f_G^A(\alpha, \lambda_A^2)$ :

$$f_G^A(\alpha, \lambda_A^2) = \delta(\alpha - \lambda_A^2), \tag{30}$$

where  $1/\lambda_A$  is an abelian correlation length of the vacuum fluctuations, it can be expressed in terms of vacuum condensates  $\lambda_A^2 = \frac{8}{9}g^2\langle \bar{u}u\rangle^2/\langle G^2\rangle \approx 0.03 GeV^2$  at  $1 GeV^2$  [20]. One comment concerning eq. (29) is that in deriving a QCD sum rule one can always perform a Wick rotation  $x_0 \to ix_0$  and treat all the coordinates as Euclidean,  $x^2 < 0$ .

Proceeding in the standard way we obtain the following sum rule:

$$m_{\eta'}^{2} f_{\eta'} f_{\eta'}^{(1)} e^{-\frac{m_{\eta'}^{2}}{M^{2}}} = M^{2} \frac{N_{f} \alpha_{s}}{4\pi} \left( \frac{1}{3\pi^{2}} \frac{N_{f} \alpha_{s}}{4\pi} M^{4} E_{2} \left( \frac{s_{0}}{M^{2}} \right) + \frac{N_{f}}{3} \left\langle \frac{\alpha_{s}}{\pi} G^{2} \right\rangle \frac{\lambda_{A}^{2}}{M^{2}} \left( 1 - \frac{\lambda_{A}^{2}}{M^{2}} \right) \right), \quad (31)$$

where

$$E_2(x) = 1 - \left(1 + x + \frac{x^2}{2}\right)e^{-x},\tag{32}$$

We stress that due to the fact that one of the gluon currents has nonzero Lorentz spin leads to the absence of direct instantons to the polarization operator of interest [21]. This property may be considered as a counterpart of the absence of the topological ghost pole in the formfactor of interest [1], while this pole does appear in the formfactor of the conserved quark-gluon current, related to the low-energy nucleon structure.

We take now the limit  $M^2 \to \infty$  and obtain the local duality relation:

$$m_{\eta'}^2 f_{\eta'} f_{\eta'}^{(1)} = \frac{N_f \alpha_s}{4\pi} \left( \frac{1}{3\pi^2} \frac{N_f \alpha_s}{4\pi} \frac{s_0^3}{6} + \frac{N_f}{3} \langle \frac{\alpha_s}{\pi} G^2 \rangle \lambda_A^2 \right), \tag{33}$$

The value of continuum threshold is found from the requirement of the most stable sum rule (31). Straightforward analysis gives us the value  $s_0 = 2.5 GeV^2$  which coincides with the one obtained in ref.[22].

Keeping the contribution due to the nonlocal gluon condensate would exceed the accuracy we are pretending to because, as it was mentioned above, we do not calculate the corresponding term in the OPE with local power corrections which are obviously small. The leading non-zero contribution coming from non-local condensate was required to analyze the stability of the sum rule and to determine the continuum threshold. However, the relative numerical value of the non-local condensate contribution is small (like that of the dropped local term), and we neglect it in what follows.

For the overlap factor  $f_{\eta'}^{(2)}$  an analogous relation taken to the same accuracy is:

$$m_{\eta'}^2 f_{\eta'} f_{\eta'}^{(2)} = -\frac{1}{420\pi^2} \left(\frac{N_f \alpha_s}{4\pi}\right)^2 \frac{s_0^3}{6}$$
 (34)

The net form for the additional term for the axial form factor at small momentum transferred looks like

$$m_N^5 \lambda_G^2 \delta G_{eff}^{(0)}(Q^2) e^{-\frac{m_N^2}{M^2}}$$

$$= \frac{2}{3} \langle \bar{u}u \rangle M^4 e^{\frac{Q^2}{4M^2}} \left\{ \frac{N_f}{(4\pi)^2} \left( \frac{\alpha_s}{\pi} \right)^2 \left[ Q^4 \ln \left( \frac{s_0 + Q^2}{Q^2} \right) - s_0 Q^2 + \frac{s_0^2}{2} \right] \right.$$

$$\left. - \frac{4}{N_f} \frac{m_{\eta'}^2 f_{\eta'}}{Q^2 + m_{\eta'}^2} \left[ f_{\eta'}^{(1)} \varphi_{\eta'}^{(1)}(1/2) + f_{\eta'}^{(2)} \dot{\varphi}_{\eta'}^{(2)}(1/2) \right] \right\}$$

$$= \frac{2}{3} \langle \bar{u}u \rangle M^4 e^{\frac{Q^2}{4M^2}} \frac{N_f}{(4\pi)^2} \left( \frac{\alpha_s}{\pi} \right)^2$$

$$\left\{ Q^4 \ln \left( \frac{s_0 + Q^2}{Q^2} \right) - s_0 Q^2 + \frac{s_0^2}{2} - \frac{s_0^3/3}{Q^2 + m_{\eta'}^2} \right\}$$
(35)

In the last line we have substitute the local duality relations for residue factors and take the asymptotical form for the wave functions

$$\varphi^{(1)}(\alpha) = 30\alpha^2 \bar{\alpha}^2.$$

$$\varphi^{(2)}(\alpha) = 420(\alpha - \bar{\alpha})\alpha^2 \bar{\alpha}^2$$
(36)

Note that in eq. (35) we have put  $\tau_1 = \tau_2 = 2M^2$  in order not to introduce the asymmetry between the initial and final states and to make contact with Borel parameter of the two-point nucleon sum rules. We can observe that at large  $Q^2$  the bilocals vanish faster than the term it is correcting  $\sim Q^2$ . Note, that although the local power correction may vanish for  $Q^2 \to 0$  the modified version of nonanalyticities  $Q^{2n} \ln Q^2$  alive in this limit.

As can be easily seen, the leading nonanalyticity in the  $\langle \bar{u}u \rangle$ -term in the correction found and in the expression for form factor cancels. It is

replaced by the combination  $s_0 + Q^2$  which is "safe" in the limit  $Q^2 \to 0$ . In the same way we may correct the other nonanalyticities. In large  $Q^2$  limit where the original OPE must be valid the bilocal corrections must be absent. As we have seen, the residues of the physical spectrum can be found from the requirement that the bilocal power corrections should vanish faster at large  $Q^2$  than the contribution they are correcting. Then, in general the correction term is given by the following equation (we omit all unnecessary constants)

$$W_{BL}^{res+cont} - W_{BL}^{PT} = \frac{s_0^n/n}{Q^2 + m_{\eta'}^2} - \int_0^{s_0} \frac{ds s^{n-1}}{s + Q^2}.$$
 (37)

Using this simple recipe one can easily modify all perturbative non-analyticities of the effective axial form factor in the small momentum transferred limit. After all of them have been corrected properly we can take the limit  $Q^2 \to 0$  and obtain the sum rule for singlet axial constant directly. As was noted in Ref. [7], one should not try to fix the parameter of continuum threshold  $\sigma_0$  from the sum rule with a new current. Therefore we take the continuum threshold usual for the sum rules involving the nucleon and consider it in the local duality limit. Combining all contribution we come to the following equation

$$m_N^5 \bar{\lambda}_G^2 G_A^{(0)} = N_f a \left(\frac{\alpha_s}{\pi}\right)^2 \left\{ \frac{7}{2^3 3} m_0^2 \frac{\sigma_0^3}{6} + \left[ \frac{1}{3} R_3 + \frac{31}{2^5} m_0^2 R_2 \right] \frac{\sigma_0^2}{2} - \left[ \frac{1}{2^2 3} R_4 + \frac{751}{2^7 3^2} m_0^2 R_3 \right] \sigma_0 + \left[ \frac{1}{2^5 3^2} R_5 + \frac{1479}{2^{10} 3^3} m_0^2 R_4 \right] \right\}$$
(38)

where

$$R_n = \left[ \frac{s_0^n}{nm_{n'}^2} - \frac{s_0^{n-1}}{n-1} \right] \tag{39}$$

and its origin was clarified above. The sum rule imply that we model the continuum by effective spectral density that includes all ones which are nonzero for s > 0.

We use the standard ITEP values of condensates rescaled to the normalization point  $\mu^2 \sim m_N^2 \sim 1 GeV^2$  with the appropriate anomalous dimensions:  $a = -(2\pi)^2 \langle \bar{u}u \rangle = 0.67 GeV^3$ ,  $m_0^2 = \langle \bar{u}g(\sigma G)u \rangle / \langle \bar{u}u \rangle = 0.65 GeV^2$ ; also, we use the overlap value  $\bar{\lambda}_G^2 = 2(2\pi)^4 \lambda_G^2 = 0.3 GeV^6$  and continuum threshold  $\sigma_0 = 2.5 GeV^2$ , providing the better accuracy of the calculation of partonic densities [23]. The value of the strong coupling constant at  $1 GeV^2$  is taken to be  $\alpha_s = 0.37$  that corresponds to  $\Lambda = 150 MeV$ . We

obtain the following numerical value of the singlet axial constant

$$G_A^{(0)}(0) = 0.2. (40)$$

Varying the parameters in the reasonable range will result in the variation of the quantity within the 50%. The main uncertainties come from the errors in estimation of the t-channel continuum threshold  $s_0$  and the overlap  $\lambda_G$  of the nucleon state with that created by the new current.

Due to the anomalous non-conservation of the singlet axial current the singlet axial constant is not a renormalization group invariant. Therefore in order to compare our prediction with the experimentally mensurable quantity, we have to evolve it from QCD sum rule scale  $\mu^2 \sim 1 GeV^2$  up to the one of EMC-SMC experiment which is  $Q^2 = 10 GeV^2$  exploiting the one-loop solution of RG equation:

$$G_A^{(0)}(0, \mathcal{Q}^2) = G_A^{(0)}(0, \mu^2) exp \left\{ \frac{\gamma_2}{4\pi\beta_0} [\alpha_s(\mathcal{Q}^2) - \alpha_s(\mu^2)] \right\}, \tag{41}$$

where the anomalous dimension  $\gamma_2 = 16N_f$  and as usual  $\beta_0 = 11 - \frac{2}{3}N_f$ . However, the sensitivity to the QCD radiative corrections is poor until very large  $Q^2$  is attained and account for them would exceed the accuracy of our estimate. Nevertheless, the value obtained are in reasonable agreement with the new world average value for the singlet axial constant.

## 4. Summary.

In summary, we have calculated the singlet axial constant in the QCD sum rule framework for the form factor type problem at small momentum transferred and find the value in good correspondence with experimental one. We should mention that in our letter [24]  $G_A^{(0)}$  was somewhat overestimated because the next-to-leading twist bilocal power corrections were not accounted for and the continuum was not properly subtracted.

In ref.[22] the pioneering attempt was undertaken to evaluate  $G_A^{(0)}(0)$  by QCD sum rules in a way similar to the calculation of the octet axial constant [25]. Due to the presence of the gluon anomaly in the induced vacuum condensates the problem differs significantly from the one for the  $G_A^{(8)}(0)$ . This feature was incorporated in the calculation but nevertheless the authors did not come to the reasonable quantitative prediction of the singlet axial constant. It was conjectured that the OPE breaks down for the

singlet axial current in the axial-nucleon-nucleon vertex. At the same, we did not observe any evidence of the divergence of the OPE in the correlator under investigation: the contribution of the highest dimension vacuum condensates unsuppressed by a number of loops is absent. From the other side, the small values of the power corrections result in the good accuracy of the local duality approach and in the strong dependence of our result on the continuum threshold.

A possible line of development would be to estimate twist-three gluon contribution into the moments of the transverse spin structure function  $g_2$  [26] and the x-dependence of the twist-two polarized gluon distribution in the nucleon. However, the latter would require an elaboration of the new procedure for separation of the large and small distances in the effective four-point correlator.

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#### Appendix A.

In this appendix we present some useful integrals needed in the additional factorization of large and small distances in the ordinary Feynman diagrams. Keeping the lowest twists contributions into the perturbative parts of the bilocal correlators we find:

$$\int \frac{d^d k}{(2\pi)^d} \frac{(x(k+\tilde{q}))^n}{[k^2-l]^r} = \frac{i(-1)^r}{(4\pi)^{\frac{d}{2}}} \frac{1}{\Gamma(r)} \left\{ (x\tilde{q})^n \frac{\Gamma(r-\frac{d}{2})}{l^{r-\frac{d}{2}}} + \frac{1}{2} C_n^2 [-x^2] (x\tilde{q})^{n-2} \frac{\Gamma(r-\frac{d}{2}-1)}{l^{r-\frac{d}{2}-1}} + \frac{3}{4} C_n^4 [-x^2]^2 (x\tilde{q})^{n-4} \frac{\Gamma(r-\frac{d}{2}-2)}{l^{r-\frac{d}{2}-2}} \right\} + O(x^6), \tag{42}$$

$$\int \frac{d^{d}k}{(2\pi)^{d}} \frac{(x(k+\tilde{q}))^{n}k_{\lambda}}{[k^{2}-l]^{r}} = \frac{i(-1)^{r-1}}{(4\pi)^{\frac{d}{2}}} \frac{x_{\lambda}}{2\Gamma(r)} \left\{ C_{n}^{1}(x\tilde{q})^{n-1} \frac{\Gamma(r-\frac{d}{2}-1)}{l^{r-\frac{d}{2}-1}} + \frac{3}{2}C_{n}^{3}[-x^{2}](x\tilde{q})^{n-3} \frac{\Gamma(r-\frac{d}{2}-2)}{l^{r-\frac{d}{2}-2}} + \frac{15}{4}C_{n}^{5}[-x^{2}]^{2}(x\tilde{q})^{n-5} \frac{\Gamma(r-\frac{d}{2}-3)}{l^{r-\frac{d}{2}-3}} \right\} + O(x^{6}),$$
(43)

$$\int \frac{d^{d}k}{(2\pi)^{d}} \frac{(x(k+\tilde{q}))^{n}(kq)k_{\lambda}}{[k^{2}-l]^{r}} = \frac{i(-1)^{r-2}}{(4\pi)^{\frac{d}{2}}} \frac{x_{\lambda}}{2\Gamma(r)} \left\{ C_{n}^{2}(xq)(x\tilde{q})^{n-2} \frac{\Gamma(r-\frac{d}{2}-2)}{l^{r-\frac{d}{2}-2}} + 3C_{n}^{4}[-x^{2}](xq)(x\tilde{q})^{n-4} \frac{\Gamma(r-\frac{d}{2}-3)}{l^{r-\frac{d}{2}-3}} + \frac{45}{4}C_{n}^{6}[-x^{2}]^{2}(xq)(x\tilde{q})^{n-6} \frac{\Gamma(r-\frac{d}{2}-4)}{l^{r-\frac{d}{2}-4}} \right\} + \frac{i(-1)^{r-1}}{(4\pi)^{\frac{d}{2}}} \frac{q_{\lambda}}{2\Gamma(r)} \left\{ (x\tilde{q})^{n} \frac{\Gamma(r-\frac{d}{2}-1)}{l^{r-\frac{d}{2}-1}} + \frac{1}{2}C_{n}^{2}[-x^{2}](x\tilde{q})^{n-2} \frac{\Gamma(r-\frac{d}{2}-2)}{l^{r-\frac{d}{2}-2}} + \frac{3}{4}C_{n}^{4}[-x^{2}]^{2}(x\tilde{q})^{n-4} \frac{\Gamma(r-\frac{d}{2}-3)}{l^{r-\frac{d}{2}-3}} \right\} + O(x^{6}), \tag{44}$$

where  $\tilde{q} = \alpha q$ , and  $C_n^m = \frac{n!}{m!(n-m)!}$  are binomial coefficients.

Using these results we obtain for the factorized diagrams in the second row in fig.1:

$$\overline{W}^{(1)} = \frac{1}{\pi^2} \frac{N_f}{3} \left(\frac{\alpha_s}{\pi}\right)^2 \langle \bar{u}u \rangle \frac{(\tau_1 \tau_2)^2}{(\tau_1 + \tau_2)^2} \left(L_4 + L_6 + \frac{1}{6}L_8\right) e^{\frac{Q^2}{(\tau_1 + \tau_2)}},\tag{45}$$

$$\overline{W}^{(2)} = \frac{1}{\pi^2} \frac{N_f}{18} \left(\frac{\alpha_s}{\pi}\right)^2 m_0^2 \langle \overline{u}u \rangle \frac{(\tau_1 \tau_2)}{(\tau_1 + \tau_2)^3} \left( [4(\tau_1 \tau_2) - (\tau_1 + \tau_2)^2] L_2 + [4(\tau_1 \tau_2) - 6(\tau_1 + \tau_2)^2] L_4 + \frac{1}{6} [4(\tau_1 \tau_2) - 15(\tau_1 + \tau_2)^2] L_6 \right) e^{\frac{Q^2}{(\tau_1 + \tau_2)}}, (46)$$

$$\overline{W}^{(3)} = \frac{1}{\pi^2} \frac{N_f}{144} \left(\frac{\alpha_s}{\pi}\right)^2 m_0^2 \langle \bar{u}u \rangle \frac{(\tau_1 \tau_2)}{(\tau_1 + \tau_2)^3} \left( [4(\tau_1 \tau_2) - (\tau_1 + \tau_2)^2] L_2 + [4(\tau_1 \tau_2) - 2(\tau_1 + \tau_2)^2] L_4 + \frac{1}{6} [4(\tau_1 \tau_2) - 3(\tau_1 + \tau_2)^2] L_6 \right) e^{\frac{Q^2}{(\tau_1 + \tau_2)}}, \quad (47)$$

$$\overline{W}^{(4)} = -\frac{1}{\pi^2} \frac{7N_f}{32} \left(\frac{\alpha_s}{\pi}\right)^2 m_0^2 \langle \bar{u}u \rangle \frac{(\tau_1 \tau_2)^2}{(\tau_1 + \tau_2)^3} \left(L_2 + L_4 + \frac{1}{6}L_6\right) e^{\frac{Q^2}{(\tau_1 + \tau_2)}}, \tag{48}$$

$$\overline{W}^{(5)} = -\frac{1}{\pi^2} \frac{N_f}{8} \left(\frac{\alpha_s}{\pi}\right)^2 m_0^2 \langle \bar{u}u \rangle \frac{(\tau_1 \tau_2)}{(\tau_1 + \tau_2)^3} \left( [2(\tau_1 \tau_2) + (\tau_1 + \tau_2)^2] L_2 + [2(\tau_1 \tau_2) + 2(\tau_1 + \tau_2)^2] L_4 + \frac{1}{6} [2(\tau_1 \tau_2) + 3(\tau_1 + \tau_2)^2] L_6 \right) e^{\frac{Q^2}{(\tau_1 + \tau_2)}}, \quad (49)$$

and

$$L_n = \left(\frac{Q^2}{\tau_1 + \tau_2}\right)^{n/2} \left[ ln\left(\frac{Q^2}{\mu^2} \frac{\tau_1 \tau_2}{(\tau_1 + \tau_2)^2}\right) - S_1(n/2 - 1) \right], \tag{50}$$

with

$$S_1(\alpha) = \psi(1+\alpha) + \gamma_E. \tag{51}$$

where we have used the  $\overline{MS}$ -scheme.

As can be easily seen the logarithmic terms in eqs. (A4 - A8) reproduce the leading non-analiticities in the expressions for ordinary diagrams contributing to the form factor at the moderately large  $Q^2$ . Therefore this perturbative "long-distance" behaviour cancels exactly in the difference of the diagrams in the first and second rows in fig.1.

## Appendix B.

Here we present the formulae which enable to represent the integrals appeared in the calculations in the form of double spectral representation.

$$\frac{(\tau_1 \tau_2)^2}{(\tau_1 + \tau_2)^2} J_{11} = \int_0^\infty ds_1 ds_2 e^{-\frac{s_1}{\tau_1} - \frac{s_2}{\tau_2}} \left[ Q^2 \frac{s_1 s_2}{R^{\frac{3}{2}}} \right], \tag{52}$$

$$\frac{(\tau_1 \tau_2)^2}{(\tau_1 + \tau_2)^2} J_{02} = \int_0^\infty ds_1 ds_2 e^{-\frac{s_1}{\tau_1} - \frac{s_2}{\tau_2}} \left[ -\frac{1}{2} \left( 1 - \frac{\sigma}{R^{\frac{1}{2}}} \right) - Q^2 \frac{s_1 s_2}{R^{\frac{3}{2}}} \right], \tag{53}$$

$$\frac{(\tau_1 \tau_2)^3}{(\tau_1 + \tau_2)^3} J_{12} = \int_0^\infty ds_1 ds_2 e^{-\frac{s_1}{\tau_1} - \frac{s_2}{\tau_2}} \left[ \frac{1}{24} Q^2 \left( 1 - \frac{\sigma}{R^{\frac{1}{2}}} \right)^2 \left( 2 + \frac{\sigma}{R^{\frac{1}{2}}} \right) \right]. \tag{54}$$

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Figure captions.

**Fig.1.** Contribution to the effective axial form factor in the QCD sum rules approach. The difference of the first and second rows defines the SD(I)-regime.

**Fig.2.** Generic form of the bilocal power corrections entering the OPE with different coefficient functions.



